SAT-based Model Checking of Train Control Software.

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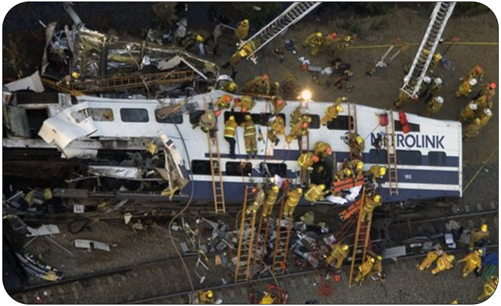
Overview

- Verification Within The Railway Domain.
- Reachable State Algorithms.
- Real World Interlockings.

Verification Within The Railway Domain

Motivation

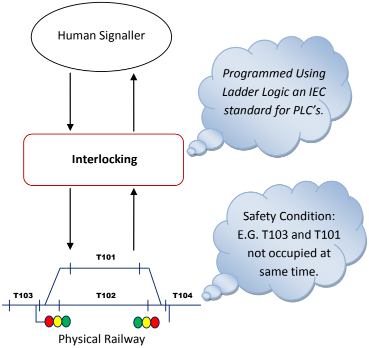
Metro-link passenger train collides with freight train.
Los Angeles – Sept 2008.



25 people killed, over 100 people injured.

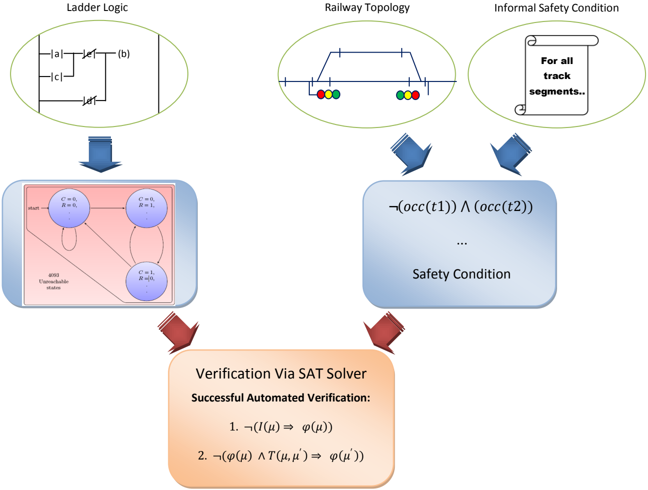
Interlockings

A major system responsible for ensuring safety within the railway is the interlocking system.



- Interlockings control aspects such as signals and points.
- Interlockings are written by Invensys in a logic similar to propositional logic.

Railway Verification in Propositional Logic – Kanso 2008



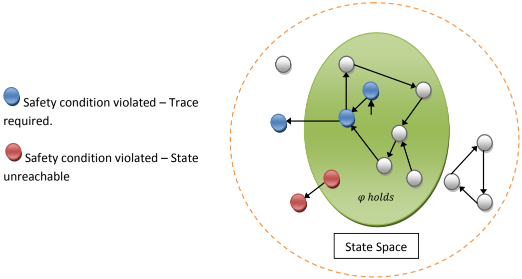
Discussion of Kanso'08

Positive:

- Successful verification of some safety properties of a real interlocking.

Problematic:

- Unclear: Is a violation reachable?



- Costly human interaction required.

Our Aims

- If a counterexample is found, produce an error trace to the counterexample.
- Devise a verification method which ignores unreachable states.
- Implement these techniques into a useable verification tool which works on real world interlockings.

Our Approach

SAT-based Model Checking

k-bounded Model Checking

$i \leftarrow 0$
$B_0 \leftarrow \{\mu \mid I(\mu)\}$
while $i \leq k$ **do**
  for $\mu \in B_i$, if $\neg(\varphi(\mu)) \in SAT$ **return** "unsafe" + trace; **stop**
  $B_{i+1} \leftarrow \{\mu' \mid T(\mu, \mu'),\ \mu \in B_i\}$
  $i \leftarrow i + 1$
return "safe"

Unbounded Model Checking

Change $i \leq k$ to $B_{i+1} \subseteq B_0 \cup \cdots \cup B_i$.

Some Definitions

Definition: Series of transitions.

We define a series of n transitions T_n in an automaton as follows:

$$T_n = \bigwedge_{0 \leq i \leq n-1} T(S_i, S_{i+1})$$

where $T(S_i, S_{i+1})$ is a transition from state S_i to state S_{i+1}.

Formula size: $O(kn)$, k number of rungs, n number of iterations.

Knowing when to stop?

Definition: Inclusion Property

We define an inclusion check as:

$$P = I_n \wedge T_{n+1} \Rightarrow (LF_n \Rightarrow \bigvee_{i \leq n+1} S_{n+1} \Leftrightarrow S_i)$$

If $P \Leftrightarrow true$ then inclusion has been reached.

Definition: Loop Freedom

An Automaton A is loop free for n transitions if the following holds:

$$LF_n = T_n \wedge \bigwedge_{0 \leq i \leq j \leq n-1} \neg(S_i = S_j)$$

Formula size: $O(kn^2)$, k number of rungs, n number of iterations.

Real World Interlockings

Real World Interlocking

Problem size:

- Ladder Logic for small train station – about 550 variables.
- 1 iteration = (approx) 1 second of run-time.

Experiments:

- Without inclusion:
  - Only 500 iterations possible due to state space explosion.
  - Verification time – 523(s), more iterations: out of memory.
- With inclusion:
  - Only 50 iterations possible due to large formulae.
  - Verification time – 652(s), more iterations: out of memory.

Slicing needed!

Program Slicing

Main Idea: Construct a program slice by removing variables/rungs which have no effect on the safety condition.

- Algorithm thanks to Fokking et al.
- New correctness statement and proof:

  consider reachable states only!

Program Slicing Example

Slicing a ladder with regard to a safety condition:

$$(tlag1 \vee tlar1) \wedge \neg(tlag1 \wedge tlar1) \wedge (tlbg1 \vee tlbr1) \wedge \neg(tlbg1 \wedge tlbr1).$$

```
while(true){
crossing1 = (req0 && ...
req1 = (pressed0 && ...
tlag1 = ((not crossing1) ...
tlbg1 = ((not crossing1) ...
tlar1 = crossing1;
tlbr1 = crossing1;
plag1 = crossing1;
plbg1 = crossing1;
plar1 = (not crossing1);
plbr1 = (not crossing1);
audio1 = crossing1;
}
```

Figure: Original Ladder

```
while(true){
crossing1 = (req0 && ...
req1 = (pressed0 && ...
tlag1 = ((not crossing1) ...
tlbg1 = ((not crossing1) ...
tlar1 = crossing1;
tlbr1 = crossing1;
}
```

Figure: Sliced Ladder

Correctness Theorem

Theorem:

Given a ladder logic program P and a safety condition φ,

$$A(P) \models \varphi \Leftrightarrow A(P|_\varphi) \models \varphi.$$

Proof Sketch: Argue on reachability of states in each automaton.

Implementation of slicing in Haskell.

Program Slicing – Some Results

Our Results on Real World Interlockings:

- Ladder with approx 550 variables reduced to ladder with 62 variables.
- Without Inclusion:
  - Up to 2000 iterations 4553(s), more iterations: out of memory.
- With Inclusion:
  - Up to 200 iteration 1554(s), more iterations: out of memory.

Underlying prover: Equinox.

Commercial Tool: about 100 iterations.

Overall Verification Results

k-bounded Model Checking:

Property	Kanso'08	k-bounded MC
φ_1	Safe	Safe
φ_2	Unsafe	Counterexample (4 iterations)
φ_3	Unsafe	Counterexample (3 iterations)
φ_4	Unsafe	Counterexample (1 iteration)

Unbounded Model Checking:

- Inclusion not reached in 200 iterations.
- Current slices: $\sim$ 60 variables.
- Experiments show: $\sim$ 30 variables work out.

Conclusion

Summary

- New slicing Theorem w.r.t. reachable states only.
- Slicing works very well to reduce formulae size.
- Verified succesfully two real interlockings:
  - For all given safety conditions we either -
    - proved safety, or
    - returned counter example.
- Open problem (with no impact to practice?): Inclusion not reached, formulae still too big.

Future Work

- Remove functional dependencies: to reduce formulae size further.
- Look at modelling using First Order logic.
- Explore compositional reasoning of ladder logic templates used by Invensys.

Thanks!

Automata Definition

Definition: Automaton

Given a ladder logic program P over $V = I \cup O \cup O'$. An automaton is a triple $(S, I, \longrightarrow)$, where

- $S = \{\nu | \nu : I \cup U \rightarrow \{0, 1\}\}$.
- $I = \{\nu' | \nu \models \neg I_{cond}, \nu \cup \nu' \models \psi_P\}$
- $\nu \longrightarrow \nu'$ iff $\nu \cup \nu' \models \psi_P$.